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MOBILE SOURCE ESTIMATION WITH AN ITERATIVE REGULARIZATION METHOD

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Abstract - In this paper, several numerical aspects of the resolution of a nonlinear ill-posed problem by a conjugate gradient method are presented. From an experimental situation, a thermal system described by a set of partial differential equations is considered. The parametric identification of an unknown moving heating source is performed by a conjugate gradient method which acts as a regularizing method. The interest of such a minimization method is shown for optimal sensor location and while the measurements are noisy disturbed.

1. INTRODUCTION

This paper is dedicated to the problem of determining the time-history of the heat flux delivered by a mobile small-size heater acting on the front face of a sheet of metal by means of the time history of measured temperatures on the rear face. It is well known that such an inverse problem is ill-posed since solution is strongly affected by data errors : initial state, measurements bias, discrete approximation, … In the following paragraph, several techniques for regularizing and solving ill-posed problems are presented. Then the experimental situation is exposed and modeled. The conjugate gradient method is proposed for the resolution of the inverse problem and leads to iterative resolution of three well-posed problems: direct problem, sensitivity problem and adjoint problem. Numerical results are exposed and the effect of disturbing noises is investigated. Moreover, it is shown that the resolution of the sensitivity equations provides a strategy for the choice of relevant sensors.

2. ILL-POSED INVERSE PROBLEMS

The problem of determining the input signal u of a dynamic system when only the output $\tilde{y}$ is known as inverse problem. Denoting by δy, the measurement error and F, the model structure, the measured data are related to the unknown inputs with the following relationship :

$$\tilde{y} = F(u) + \delta y \tag{1}$$

For the system studied hereafter, both the structure and the parameters values of the physical model are known prior to the measurement, contrariwise to blind inversion where the parameter values are also unknown. It is well-known that inverse problems are ill-posed. A problem is well-posed if it satisfies the three Hadamard conditions of existence, uniqueness and stability. If any of the previous conditions are not satisfied, then the problem is ill-posed. For continuous diffusive systems such the thermal process studied in this paper, the inverse operator is unbounded and the presence of measurement noise in the actual data makes the problem instable ; the inverse problem is then ill-posed. Overcoming the ill-posedness of inverse problem is known as regularisation. The key issue in solving inverse problems, is how to introduce just enough prior information to obtain a satisfactory result [4]. Several techniques for regularising and solving ill-posed problems have been proposed and used. One first simple idea consisted on choosing a restricted class of inputs : steps, band limited signals, polynomial bases … A more attractive technique, however, does not use constraint on the input signal structure but build a regularisation operator depending on a certain parameter χ, called the regularisation parameter.

$$\hat{u} = R(\delta y, \chi) = \arg\min J_\chi(u) \tag{2}$$

The family of operator yields a correct solution to the problem when both the regularisation parameter and the measurement errors tends to zero.

$$\forall \delta y,\ \forall \varepsilon > 0, \exists \chi(\delta y), \exists \alpha(\varepsilon)\ \mid \left\|y^* - \tilde{y}\right\|^2 \le \alpha(\varepsilon) \Rightarrow \left\|u^* - \hat{u}\right\|^2 \le \varepsilon \tag{3}$$

Tikhonov first suggested the use a smoothing function to account for prior information. The criterion used for identification writes then

$$J_\lambda(u) = \left\|\tilde{y} - F(u)\right\|^2 + \chi \cdot \Omega(u) \tag{4}$$

where the smoothing function Ω(.) uses the derivative of the unknown input variable

$$\Omega(u) = \sum_{k=0}^{N} \alpha_k \cdot \left\|u^{(k)}\right\|^2 \tag{5}$$

In eqn. (4), the determination of the optimal value for the smoothing parameter remains an open problem. A commonly used procedure is based on the Morozov's discrepancy principle [5] and the references therein) : assuming that a bound δy on measurement errors statistics is known, the smoothing parameter χ^* is chosen such that

$$\left\| F(\hat{u}) - \tilde{y} \right\|^2 = \delta y \tag{6}$$

with
$$\hat{u} = \arg\min J_{\chi^*}(u) \tag{7}$$

The use of the function (5) as prior information is not always convenient, for instance when the searched signal involves peaks. In such cases, a Bayesian inference is preferred and the regularised solution minimises a criterion of the following form

$$J(u) = D_1(\tilde{y}, F(u)) + D_2(u, u_0) \tag{8}$$

where the functionals D_1 and D_2 are derived from the statistical prior laws on measurement errors and the unknown parameters. In practice, the maximum entropy principle makes it possible to translate any prior knowledge on the unknown parameters and measurement noise to the probabilistic laws used in eqn. (8), see [4].

In the context of inverse heat conduction problems, an alternate regularisation procedure has been introduced in [2] : the iterative regularisation scheme with the gradient methods. The gradient methods are non-linear minimisation algorithms based on a limited expansion of the cost : they include the gradient method, the Newton, the Gauss-Newton and conjugate-gradient methods. The procedure consists on the minimisation of the following criterion based on the model-data discrepancy and where there is no smoothing function

$$J(u) = \left\| \tilde{y} - F(u) \right\|^2 \tag{9}$$

In order to minimize criterion (9), the gradient methods generate a series of solutions that satisfies the equation

$$u_{n+1} = u_n - \beta_n d_n \tag{10}$$

The following properties for the *gradient* methods are given in [2]:

Property 1 : When there is no measurement noise and the model used is correct, the series (10) converges to the true solution u^*.

Property 2 : When the data are corrupted with the noise δy , the following property is satisfied

$$\forall \delta y \quad \forall \varepsilon > 0 \quad \exists N \;\; u_N \text{ is stable and } \lim_{\delta y \to 0} \left\| u^* - u_N \right\|^2 \to 0 \tag{11}$$

Equation (11) shows that there exists an iteration number N for which the approximate solution u_N is regularised. The iteration number acts then as the smoothing parameter of equation (4).

Property 3 : Some implicit formulas have been introduced in [2] in order to compute the optimal iteration number as a function of measurement noise and model error $N(\delta y, \delta F)$. In fact, the actual errors remaining insufficiently known, the following heuristic is used : run iterations until the decrease of the criterion (9) becomes insignificant. As shown in Figure 1, the optimal iteration number is N_ν .

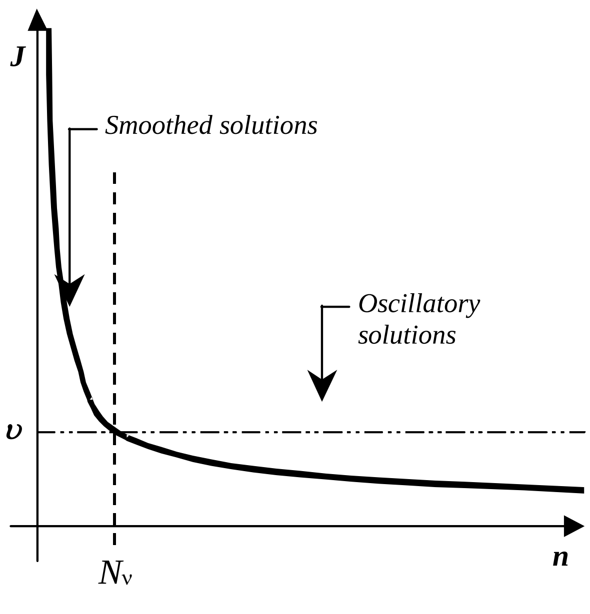

Figure 1. Optimal iteration number in iterative regularisation procedure.

In the next sections, the iterative regularisation procedure is used with the conjugate-gradient method, in order to solve the inverse heat conduction problem.

3. EXPERIMENTAL SITUATION AND DIRECT PROBLEM

The experimental apparatus presented in this communication has been developed in order to investigate several applications, such as optimal control for welding processes, hardening of steel due to the thermal shock induced by high density of solar flux, optimal experiment design for tribometer. A circular moving heating source is moved in the horizontal plane, very closely to the underneath sheet of metal. The steel selected is a refractory NS30 steel, whose thermophysical properties are given in the following. The motion is obtained with accurate two step-by-step motors. The spatial uniformity of the temperature heat source is controlled by mean of a water circulation around its support. In order to describe the temperature evolution of the sheet of metal heated by the circular source, a model is established.

Let us denote by :

- $x \in \Omega$, the space variable, where $\Omega \subset \mathbb{R}^3$ is the domain corresponding to the parallelepipedic sheet of metal. The surface of Ω is Γ. $\Omega = \{x = (x_1, x_2, x_3) \in [0;0.3]\times[0;0.2]\times[0;5\,10^{-3}]\}$
- $t \in T = [0, t_f]$ is the time variable, $t_f = 600s$
- $\theta(x,t)$ is the temperature and the initial temperature is constant : $\theta_0 = 293K$
- $\rho(\theta)$ the mass density $(kg.m^{-3})$, $\rho(\theta) = -0.444\theta + 8121.3$
- $c_p(\theta)$ the specific heat $(J.kg^{-1}.K^{-1})$, $c_p(\theta) = \begin{cases} 0.22\theta + 432.7 & if \quad 273 \le \theta \le 888 \\ 0.46\theta + 219.6 & if \quad 888 < \theta \le 1300 \end{cases}$
- $\lambda(\theta)$ the thermal conductivity $(W.m^{-1}.K^{-1})$, $\lambda(\theta) = 0.0129\theta + 10.03$
- h the convective exchange coefficient which is quite difficult to estimate. For natural convection phenomena, realistic values are proposed on the boundaries,
- ε the emissivity, is considered equal to 1 (while the surface of the material is black painted),
- σ the Stefan constant $\sigma = 5.67\,10^{-8} \quad W.m^{-2}.K^{-4}$,
- $\omega_s(t) \subset \Gamma$ is the sub-domain of Γ corresponding to the spatial support of the circular heating source. The heating source is a disk on the upper face of the sheet of steel, denoted $D(I(t),r)$, of center $I = [a\cos(\omega t + \phi) \;\; b\sin(\omega t + \phi) \;\; 0.005]^T$ and radius $r = 0.01\,m$. Then ω_s is formulated as follows : $\omega_s = \{x = (x_1, x_2, 0.005) \;\; |(x_1, x_2) \in D(I(t), r)\}$. Several trajectories are studied with the previous formulation : non moving, rectilinear, semi-circular, semi-ellipsoid. a, b, ω, ϕ are given.
- $\varphi(t)$ the heat flux, taken constant on $D(I(t),r)$.

The thermal evolution of the material during the process is described by the following equations :

- state equation :

$$\forall(x,t) \in \Omega \times T, \quad \rho(\theta)c_p(\theta)\frac{\partial\theta}{\partial t} - div\left(\lambda(\theta)\overrightarrow{grad}(\theta)\right) = 0 \tag{12}$$

- initial condition :

$$\forall x \in \Omega, \quad \theta(x,0) = \theta_0 \tag{13}$$

- heating condition :

$$\forall(x,t) \in \omega_s \times T, \quad -\lambda(\theta)\frac{\partial\theta}{\partial\vec{n}} = -\varphi(t) \tag{14}$$

- heat exchange condition :

$$\forall(x,t) \in (\Gamma - \omega_s) \times T, \quad -\lambda(\theta)\frac{\partial\theta}{\partial\vec{n}} = h(\theta - \theta_0) + \varepsilon\sigma(\theta^4 - \theta_0^4) \tag{15}$$

where $\vec{n}$ is the normal vector exterior to the surface.

According to the previous notations, a nonlinear distributed parameter system (DPS) is considered and the direct problem can be formulated as follows :

__Problem__ P_{dir} : find the temperature $\theta(x,t)$ solution of the nonlinear DPS $\{S\}$ defined by eqns (12)-(15).

Problem P_{dir} is solved by a finite element method in space and finite differentiation in time.

4. RESOLUTION OF THE INVERSE PROBLEM

In the following, the resolution of the inverse problem consists in estimating the unknown magnitude $\varphi(t)$ of the heat source while its trajectory and motion speed are known.

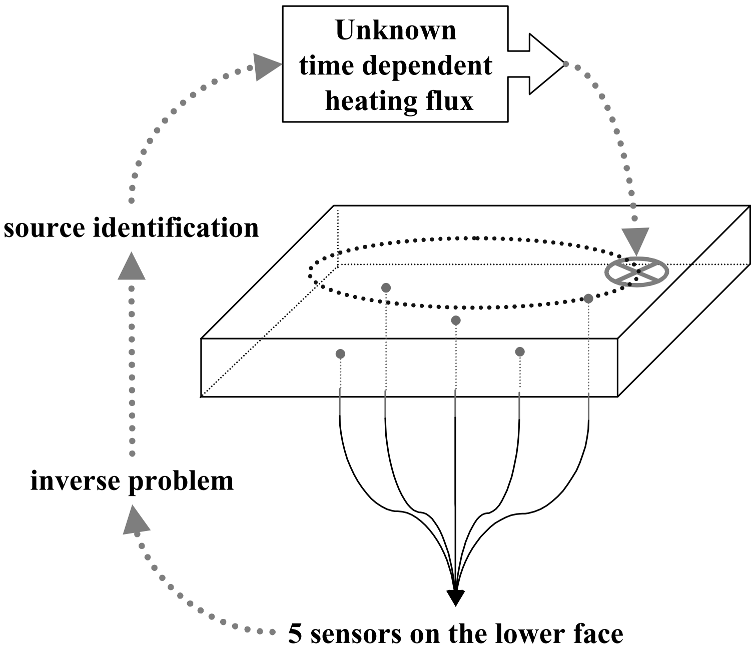

Figure 2. Inverse problem.

Heat flux is modeled as a time-varying function, by

$$\varphi(t) = \sum_{i=1}^{N-1} \varphi_i \xi_i(t) \tag{16}$$

In the latter, $\xi_i(.)$ is a time-dependent continuous piecewise linear function such that, for $t_i = 600j/N$

$$\text{for } j = 1,\cdots,N\ , \quad \xi_i(t_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{17}$$

According to this notation, $\varphi(0) = \varphi(600) = 0$ W.m^{-2} and $\varphi(t)$ is fully known when the coefficients φ_i, for i=1 to N-1 are identified.

Then the following inverse problem is considered :

Problem P_{inv} : find $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, ..., \varphi_{N-1}]^T$, which minimizes the cost function :

$$J(\boldsymbol{\varphi}) = \frac{1}{2}\int_T \left(\sum_{s=1}^{5} \left(\theta(x_s, t; \boldsymbol{\varphi}) - \hat{\theta}_s(t) \right)^2 \right) dt \tag{18}$$

where θ_s is the temperature measured at sensor s ($s = 1,\cdots,5$) located on point x_s ; with the constraint $\{\theta(x,t)$ is solution of (S)$\}$.

4.1 General conjugate gradient algorithm

The general conjugate gradient algorithm is as follows [1]:

Initialize: $k = 0$,

$\boldsymbol{\varphi}^0$: initial approximation of $\boldsymbol{\varphi}$,

$\mathbf{d}^0 = -\nabla J(\boldsymbol{\varphi}^0) = -(\partial J/\partial \varphi_i)_{i=1,\cdots,N-1}(\boldsymbol{\varphi}^0)$: initial descent direction,

At iteration k : obtain next point and next descent direction, from point $\boldsymbol{\varphi}^k$ as follows :

$$\gamma^k = \arg\min_{\gamma \in \mathbb{R}} J(\boldsymbol{\varphi}^k + \gamma \mathbf{d}^k)$$

$$\boldsymbol{\varphi}^{k+1} = \boldsymbol{\varphi}^k + \gamma^k \mathbf{d}^k$$

$$\beta^k = \left\| \nabla J(\boldsymbol{\varphi}^{k+1}) \right\|^2 / \left\| \nabla J(\boldsymbol{\varphi}^k) \right\|^2 .$$

$$\mathbf{d}^{k+1} = -\nabla J(\boldsymbol{\varphi}^{k+1}) + \beta^k \mathbf{d}^k$$

Stop the iterative process if $J(\boldsymbol{\varphi}^{k+1})$ has reached the admissible level of minimization.

Several numerical aspects have to be carefully investigated at each iteration k :

- estimation of the cost-function : resolution of the direct problem (heat flux φ^k is taken into account) in order to estimate $\theta(x_s,t;\varphi)$.
- estimation of the gradient of the cost-function : resolution of the adjoint problem in order to estimate $\nabla J(\varphi^k)$ and the descent direction.
- estimation of the descent depth : resolution of the sensitivity problem in the descent direction.

4.2 Adjoint problem for the gradient calculation

The gradient $\nabla J(\varphi)$ verifies :

$$\begin{aligned}\delta J &= \sum_{i=1}^{N-1}\left(\frac{\partial J(.)}{\partial \varphi_i}\delta\varphi_i\right)\\ &= J(\varphi+\delta\varphi)-J(\varphi)\\ &= \int_T\left(\sum_{j=1}^{5}\left(\theta(x_j,t;\varphi)-\hat{\theta}_j(t)\right)\delta\theta(x_j,t;\varphi)\right)dt\\ &= \int_T\int_\Omega\left(\sum_{j=1}^{5}\left(\theta(x_j,t;\varphi)-\hat{\theta}_j(t)\right)\zeta_j(.)\right)\delta\theta d\Omega dt\end{aligned} \tag{19}$$

where $\zeta_j(x)$ is the Dirac distribution of sensor j.

Let $L(\theta,\varphi,\psi)$ the Lagrangian associated to the minimization of the functional defined in eqn. (18) with constraints (S) :

$$L(\theta,\varphi,\psi)=J(\theta,\varphi)+\left\langle\psi,\rho(\theta)c(\theta)\frac{\partial\theta}{\partial t}-div\left(\lambda(\theta)\overrightarrow{grad\theta}\right)\right\rangle \tag{20}$$

where $\psi(x,t)$ is a Lagrange multiplier and $\langle u,v\rangle$ is the scalar product in $L^2(T,L^2(\Omega))$.

When ψ is fixed then

$$\delta L=\frac{\partial L}{\partial\theta}\delta\theta+\frac{\partial L}{\partial\varphi}\delta\varphi \tag{21}$$

The Lagrange multiplier $\psi(x,t)$ is chosen such that:

$$\forall\delta\theta,\ \frac{\partial L}{\partial\theta}\delta\theta=0 \tag{22}$$

Then according to the expression developed in [1], $\psi(x,t)$ has to satisfy the following equations:

- state equation:

$$\forall(x,t)\in\Omega\times T\,,\quad -\rho(\theta)c(\theta)\frac{\partial\psi}{\partial t}-\lambda(\theta)\Delta\psi=-\sum_{j=1}^{5}\left(\theta(x_j,t;\bar{\alpha})-\hat{\theta}_j(t)\right)\zeta_j(.) \tag{23}$$

- final condition:

$$\forall x\in\Omega\,,\quad \psi(x,t_f)=0 \tag{24}$$

- boundary condition:

$$\forall(x,t)\in\omega_s\times T\,,\quad -\lambda(\theta)\delta\theta\frac{\partial\psi}{\partial\vec{n}}=-\psi\delta\varphi(t) \tag{25}$$

- boundary condition:

$$\forall(x,t)\in(\Gamma-\omega_s)\times T\,,\quad -\lambda(\theta)\frac{\partial\psi}{\partial\vec{n}}=\psi\left(h+\varepsilon\sigma4\theta^3\right) \tag{26}$$

In order to determine the Lagrangian multiplier $\psi(x,t)$, the following adjoint problem is solved

<u>*Problem*</u> P_{lag} *: find the Lagrangian multiplier* $\psi(x,t)$ *solution of the DPS* $\{S_{lag}\}$ *defined by eqns* (23)-(26).

Considering $\psi(x,t)$ solution of $\{S_{lag}\}$ and $\theta(x,t)$ solution of $\{S\}$, it becomes :

$$\delta J=\delta L \tag{27}$$

$$\sum_{i=1}^{N-1}\left(\frac{\partial J(.)}{\partial\varphi_i}\delta\varphi_i\right)=-\int_T\int_{\omega_s}\psi\delta\varphi dt d\Gamma \tag{28}$$

Thus :
$$\frac{\partial J(.)}{\partial \varphi_i} = -\int_T \psi \frac{\partial \varphi}{\partial \varphi_i} dt d\Gamma \tag{29}$$

4.3 Calculation of the descent depth

The determination of the descent depth γ^k in the conjugate gradient algorithm is obtained by minimizing the criterion:

$$\frac{1}{2}\int_T \left(\sum_{j=1}^{5} \left(\theta\left(x_j, t; \boldsymbol{\varphi}^k + \gamma^k \mathbf{d}^k\right) - \hat{\theta}_j(t) \right)^2 \right) dt \tag{30}$$

The solution of (30) is given by

$$\hat{\gamma}^k = \int_T \left(\sum_{j=1}^{5} \delta\theta\left(x_j, t; \boldsymbol{\varphi}^k\right) \left(\theta\left(x_j, t; \boldsymbol{\varphi}^k\right) - \hat{\theta}_j(t) \right) \right) dt \Bigg/ \int_T \left(\sum_{j=1}^{5} \left(\delta\theta\left(x_j, t; \boldsymbol{\varphi}^k\right) \right)^2 \right) dt \tag{31}$$

where $(\delta\theta)_j\left(x_j, t; \boldsymbol{\varphi}^k\right)$ is the solution of the sensitivity problem P_{sens} in the direction $\delta\boldsymbol{\varphi} = \mathbf{d}^k$.

In order to determine the temperature variation resulting from a heat flux variation $\mu\delta\boldsymbol{\varphi}$, where μ is a scalar, we derive the following sensitivity equations, where the sensitivity function is defined as follows:

$$\delta\theta(x, t; \boldsymbol{\varphi}) = \lim_{\mu \to 0} \frac{\theta(x, t; \boldsymbol{\varphi} + \mu\delta\boldsymbol{\varphi}) - \theta(x, t; \boldsymbol{\varphi})}{\mu} \tag{32}$$

Denote by : $\theta^+ = \theta(x, t; \boldsymbol{\varphi} + \mu\delta\boldsymbol{\varphi})$, $\theta = \theta(x, t; \boldsymbol{\varphi})$ and $a(.) = \rho(.) c_p(.)$. The evolution of θ^+ is described by the following equations:

- state equation:

$$\forall (x,t) \in \Omega \times T, \quad a(\theta^+) \frac{\partial \theta^+}{\partial t} - div\left(\lambda(\theta^+) \overrightarrow{grad}(\theta^+) \right) = 0 \tag{33}$$

- initial condition:

$$\forall x \in \Omega, \quad \theta^+(x, 0) = \theta_0 \tag{34}$$

- heating condition:

$$\forall (x,t) \in \omega_s \times T, \quad -\lambda(\theta^+) \frac{\partial \theta^+}{\partial \vec{n}} = -(\boldsymbol{\varphi} + \mu\delta\boldsymbol{\varphi})(t) \tag{35}$$

- heat exchange condition:

$$\forall (x,t) \in (\Gamma - \omega_s) \times T, \quad -\lambda(\theta^+) \frac{\partial \theta^+}{\partial \vec{n}} = h(\theta^+ - \theta_0) + \varepsilon\sigma\left(\theta^{+4} - \theta_0^4\right) \tag{36}$$

By comparison between eqns (12)-(15) and eqns (33)-(36), the following equations are obtained :

- state equation:

$$\forall (x,t) \in \Omega \times T, \quad \frac{\partial}{\partial t}\left(a(\theta)\delta\theta \right) - \Delta\left(\lambda(\theta)\delta\theta \right) = 0 \tag{37}$$

- initial condition:

$$\forall x \in \Omega, \quad \delta\theta = 0 \tag{38}$$

- heating condition:

$$\forall (x,t) \in \omega_s \times T, \quad -\frac{\partial}{\partial \vec{n}}\left(\lambda(\theta)\delta\theta \right) = -\delta\boldsymbol{\varphi} \tag{39}$$

- heat exchange condition:

$$\forall (x,t) \in (\Gamma - \omega_s) \times T, \quad -\frac{\partial}{\partial \vec{n}}\left(\lambda(\theta)\delta\theta \right) = h\delta\theta + 4\varepsilon\sigma\theta^3\delta\theta \tag{40}$$

According to the previous notations, the sensitivity problem can be formulated as follows :

<u>*Problem*</u> P_{sens} *: find the temperature variation* $\delta\theta(x,t)$ *solution of the linear DPS* $\{Ssens\}$ *defined by equations* (37)-(40) *for given* $\boldsymbol{\varphi}$ *,* θ *and* $\delta\boldsymbol{\varphi}$.

Problems P_{lag} and P_{sens} are solved by the same numerical method which is implemented for P_{dir}.

5. NUMERICAL RESULTS

In order to show the performances of the conjugate gradient method for the resolution of the previous nonlinear ill-posed problem, a numerical experiment with a given heat flux is considered and this leads to simulated measurements (see Figure 3). Disturbing noises are added to the simulated measurements shown on Figure 3.

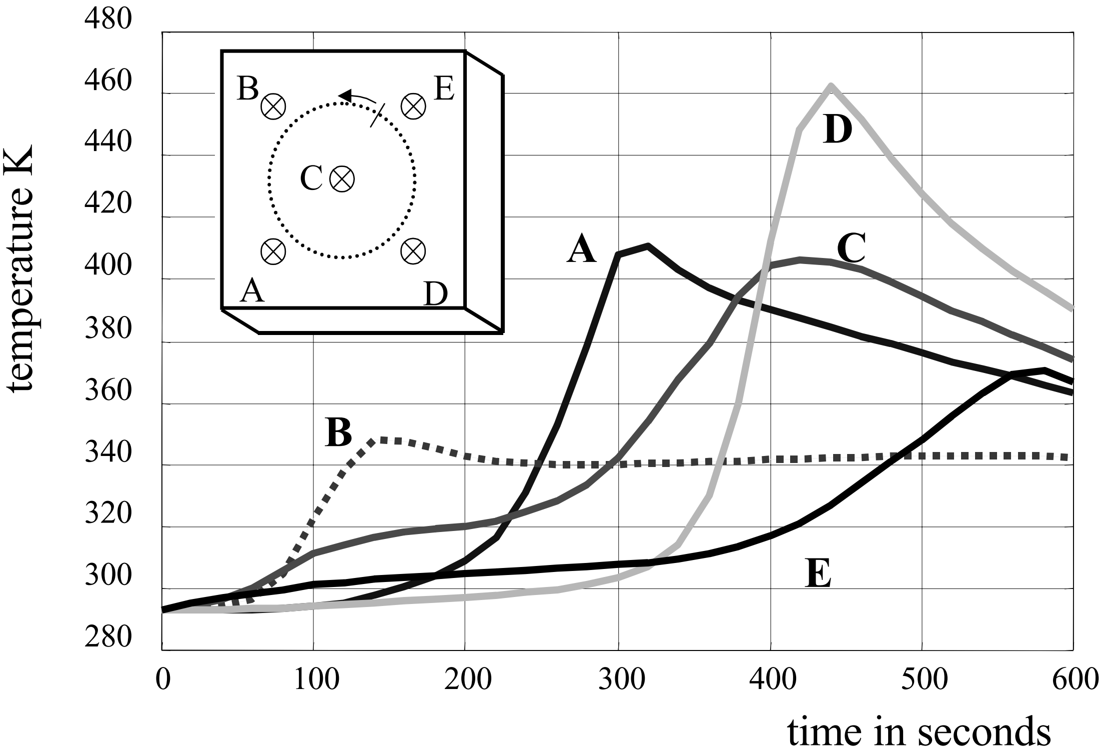

Figure 3. Simulated measurements.

5.1 Regularizing effect of the conjugate gradient method

In Figure 4, the evolution of the cost-function is presented : black line (without disturbing noises on the measurements), dashed line (with noisy disturbed measurements). Identification results are presented in Figure 5.

- without disturbing noises on the measurements, numerical solution is shown at iteration 100,
- with noisy disturbed measurements, numerical solution is shown at the admissible level of minimization (black line) and at iteration 100.

Remarks :

- From Figures 4 and 5 it can be seen that the conjugate gradient algorithm is an efficient regularization method.
- Without disturbing noises on the measurements, the cost-function is well decreasing towards the good solution.
- With noisy disturbed measurements, even if the cost-function is still decreasing, it is important to stop the algorithm at the admissible level of minimization in order to ensure stability for the estimated heat fluxes.

5.2 Choice of the most relevant sensors

Since the admissible level of minimization depends on the number of noisy disturbed measurements, the effect of disturbing noises can be reduced by taking into account less observations. In order to choose the most relevant sensors at each instant, the sensitivity functions can be considered.

At each iteration of the minimization algorithm, the resolution of the sensitivity problem in the descent direction leads to the determination of the sensitivity functions. For example, in Figure 6, sensitivity function after the first iteration are shown. Thus, after the first iteration, most relevant sensors can be chosen. For example: for $t \in [0,60]$, the most sensitive sensor is E, for ($t \in [60,220] \to$B), ($t \in [220,360] \to$A), ($t \in [360,520] \to$D) ($t \in [540,600]$ $\to$E). Such strategy has to be estimated at each iteration of the minimization algorithm and can lead to the choice of one or several sensors which provide the relevant observations of the state of the system.

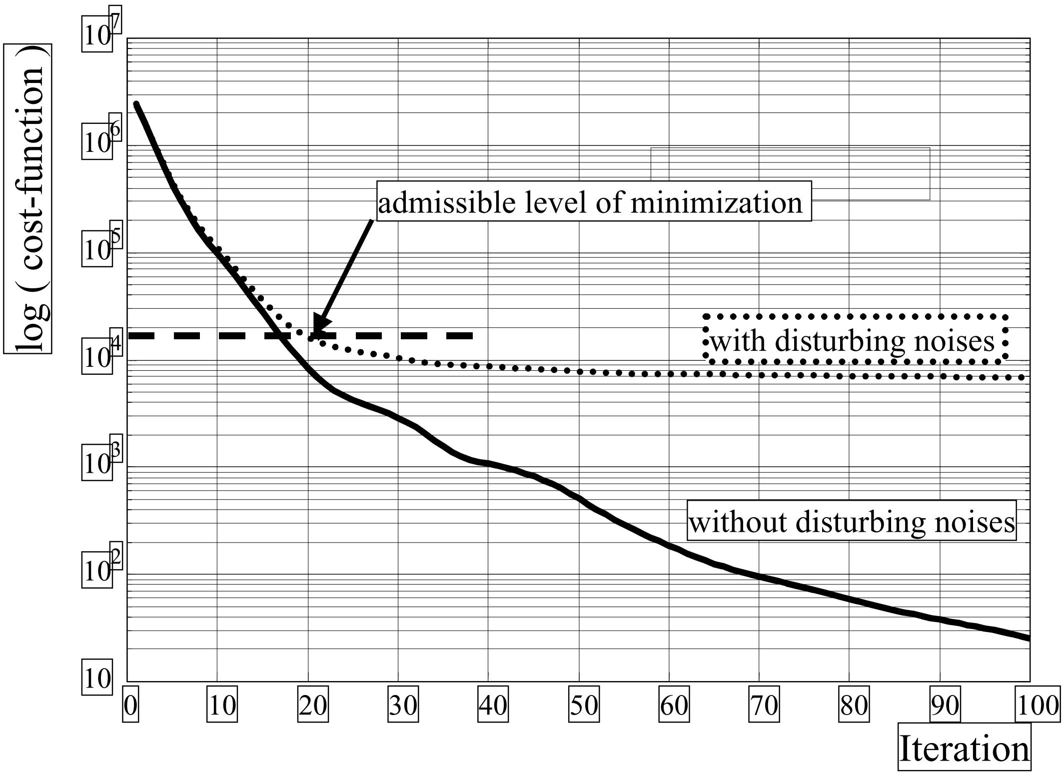

Figure 4. Identification with noisy disturbed data.

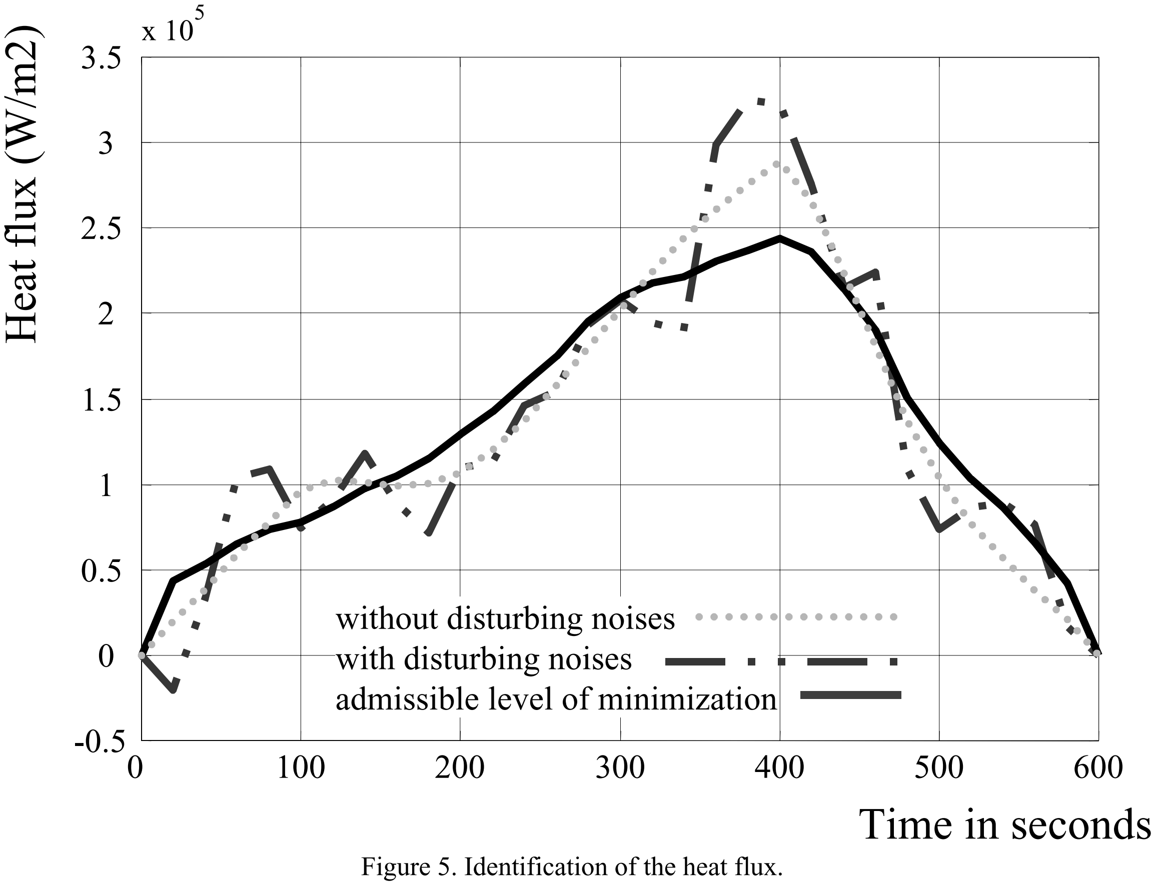

Figure 5. Identification of the heat flux.

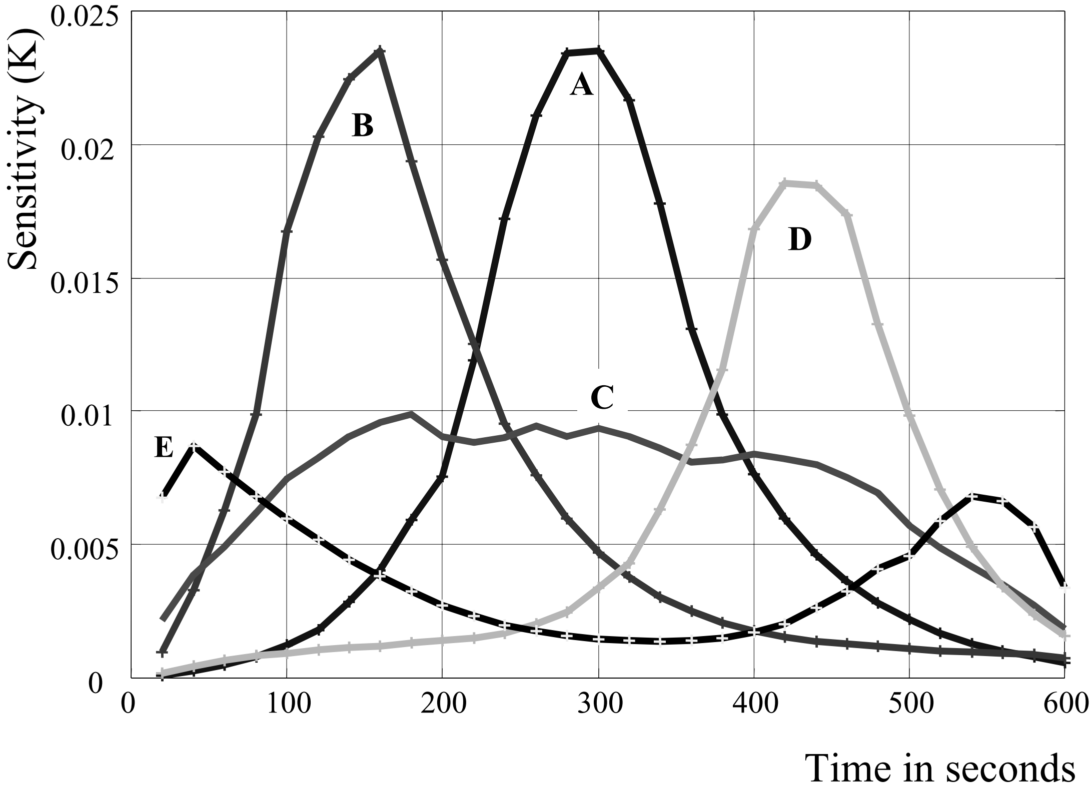

Figure 6. Sensitivity at iteration 1.

6. CONCLUSIONS

In this communication, we have investigated the determination of the time history of a moving heat flux on the upper surface of a metal sheet from temperature measurements on the rear surface via an iterative regularization method with the conjugate gradients. With numerical simulations obtained from the resolution of a partial differential equations in a three-dimensional geometry, we have shown the good performances of the conjugate gradient algorithm in such a complex ill-posed inverse problem while assuming noisy measurements. In addition, analysis of sensitivity functions at each step of the minimization algorithm leads to a correct strategy for system observations. Finally, a control loop can be implemented in order to avoid superposition of observations. As the continuation of this work, uncertainty on the sensor location might be taken into account as a nuisance parameter. Then an optimality criterion can be defined in order to estimate the unknown heat flux without the determination of the real position of the sensor [6]. Such an approach, which seems to provide an attractive alternative, has to be carefully investigated. The next issue is to find out what should be the sampling time interval. Its determination has to be connected to the speed of motion of the source which can be an unknown parameter for the inverse problem.

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